Homework 3

Due Friday, February 28th, by end of day via Canvas

PUBH 8442: Bayes Decision Theory and Data Analysis

1. *Coke bottling!*
   Consider the model for bottling machine calibration introduced in class:

   - Given its calibration μ, a machine fills each bottle with known error variance 0.05 oz.: $y \sim \text{Normal}(\mu, 0.05)$.
   - Machine calibrations are distributed as $\mu \sim \text{Normal}(12, 0.01)$.

   The company would like to ensure that machine calibrations are above 11.9 oz. They have the option of re-calibrating a machine to $\mu = 12$ oz. (R), or not re-calibrating (N). Re-calibration costs \$500; if a machine is not re-calibrated the assigned cost is \$1000 if $\mu < 11.9$ and \$0 otherwise. If n bottles with volumes $\mathbf{y} = y_1, \ldots, y_n$ are sampled from a given machine, what is the Bayes decision rule $d(\mathbf{y}) : \mathbb{R}^n \to \{R, N\}$?

2. *IQs*
   An individual takes two IQ tests with scores centered on their true IQ μ and with error variance 64: $y_1, y_2 \overset{iid}{\sim} \text{Normal}(\mu, 64)$. Consider two models:

   $$M_1 : \mu \sim \text{Normal}(100, 225).$$

   and

   $$M_2 : p(\mu) = 1 \quad \text{for } \mu \in \mathbb{R}.$$

   Individuals are to be labeled "outliers" if the typical population model for IQ (M_1) is not appropriate and a non-informative model (M_2) would be better.

   - Note that model M_2 is improper. What is the partial Bayes factor for M_1 over M_2, $BF(y_2|y_1)$?
   - Assuming $y_1 = 147$ and $y_2 = 153$, calculate the arithmetic intrinsic Bayes factor for M_1 over M_2. Based on this result, what do you conclude for this individual?

3. Observe $Y \sim \text{Binomial}(n, \theta)$ and consider two models:

   $$M_1 : \theta = 1/2$$

   and

   $$M_2 : \theta \sim \text{Uniform}(0, 1).$$

   (a) What is the Bayes factor, BF, of M_1 over M_2 (write as a function of n and Y).

   (b) Consider a third model $M_3 : \theta \sim \text{Beta}(\alpha, \alpha)$. Write the Bayes factor, BF^*, of M_3 over M_2 as a function of n, Y, and α.

   (c) What is BF^* when $\alpha = 1$? When $\alpha \to \infty$?

   For 3(c) you may find it useful to use $B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, and the relation $\Gamma(\alpha + x) \approx \Gamma(\alpha)\alpha^x$ for $x \in \mathbb{R}$ as $\alpha \to \infty$.