Homework 6

Due by end of day Monday, 4/15

PUBH 8442: Bayes Decision Theory and Data Analysis

Include any code used to generate answers at the end of your assignment, and submit electronically via Canvas. NOTE: Because it is only one question, the score for this homework will be weighted only 50% relative to other homeworks.

1. Mining Safety (*Modified from Exercise* 3.9 *in Carlin & Louis*)

   Consider the count of coal mining disasters in England from 1851 to 1962, available at http://ericfrazerlock.com/coal_data.txt. We will use a hierarchical Poisson model for the number of disasters Y_i during year i, with a "changepoint" at year 1890:

$$Y_i \sim \begin{cases} \text{Poisson}(\theta) & \text{for } i = 1851, \ldots, 1890 \\ \text{Poisson}(\lambda) & \text{for } i = 1891, \ldots, 1962, \end{cases}$$

$$\theta \sim \text{Gamma}(1/2, b_1),\ \lambda \sim \text{Gamma}(1/2, b_2),\ \theta \text{ and } \lambda \text{ independent;}$$

$$b_1 \sim \text{Gamma}(1, 1),\ b_2 \sim \text{Gamma}(1, 1),\ b_1 \text{ and } b_2 \text{ independent.}$$

   (a) Let $\mathbf{Y} = (Y_{1851}, \ldots, Y_{1962})$. Derive the full conditional distributions for θ, λ, b_1, b_2: $p(\theta \mid \lambda, b_1, b_2, \mathbf{Y})$, $p(\lambda \mid \theta, b_1, b_2, \mathbf{Y})$, $p(b_1 \mid \theta, \lambda, b_2, \mathbf{Y})$, and $p(b_2 \mid \theta, \lambda, b_1, \mathbf{Y})$. (Hint: they will all be Gamma distributions.)

   (b) Based on part (a), describe a Gibbs sampling algorithm to simulate from the joint posterior distribution $p(\theta, \lambda, b_1, b_2 \mid \mathbf{Y})$.

   (c) Implement your Gibbs sampler in R, and use histograms of the output to approximate the marginal posterior densities for θ, λ, and $R = \theta/\lambda$.

   (d) Compute a 95% credible interval for $R = \theta/\lambda$, the ratio in mining disaster rate pre-1890 vs post-1890.

   NOTE: Here the second parameter of the Gamma distribution defines the *rate*, as in class. In the Carlin & Louis book, the second parameter defines the *scale*, which is $1/rate$.